Conformal Bootstrap: a dream come true

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Ubiquitous CFT's

The study of asymptotic behaviors plays a central role in QFT, especially IR fixed points (universality)

- All examples where the IR behavior is known correspond to conformal invariant fixed points.
- In 4D, if perturbative, a fixed point is a CFT
- If non-perturbative, no formal proof but conformality largely accepted.
- In 2D, scale invariance implies conformal invariance

A large class of physically interesting IR fixed points are:

- non-supersymmetric
- non-perturbative
- small-N

No need to hunt for such a model... write the simplest (non-free) QFT:

$$\frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\phi^2 + \frac{1}{4!}\phi^4, \qquad \text{in } 2 \leq D < 4$$

How do we describe the properties of the Wilson-Fisher fixed point, say in 3D?

Is it stable? (namely, are there relevant operators singlet under global symmetries?)

Goal of conformal bootstrap

To develop a systematic and rigorous method to study the properties of conformal invariant fixed points

Outline

1. What is a CFT?
2. Conformal bootstrap
3. An application: 3D ising model

Conformal Algebra

In D dimensions : $M_{\mu\nu}\,, P_\rho, D\,, K_\sigma \simeq SO(D|2)$

Irreducible representations of Conformal Algebra:

- infinite towers of states (or operators) with increasing, equally spaced, dimensions.
- Lower state is called Primary:

$$\mathcal{O}_{\Delta,\ell}: \begin{array}{ll} \Delta & \text{dimension} \\ \ell & \text{spin} \end{array}$$

- Other states, called Descendants, obtained applying P_μ
- representation totally characterized by scaling dimension and spin of the primary

Completeness of the Hilbert space of states $\Leftrightarrow$ OPE:

$$\mathcal{O}_{\Delta_1}(x) \times \mathcal{O}_{\Delta_2}(y) = \frac{1}{|x-y|^{\Delta_1+\Delta_2}} \sum_{\mathcal{O}} C_{12\mathcal{O}} \underbrace{\left(C_{\mu_1\ldots\mu_\ell}(y) O_{\Delta}^{\mu_1\ldots\mu_\ell}(y) + \text{descendants}\right)}_{\text{fixed by conformal symmetry}}$$

$C_{12\mathcal{O}}$ are called OPE coefficients

The power of conformal invariance

Two point function of primaries: completely fixed

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{\delta_{ij}}{x_{12}^{2\Delta_i}} \qquad x_{12} \equiv |x_1 - x_2| \qquad \Delta_i = [\mathcal{O}_i]$$

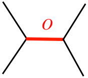
Three point function of primaries: fixed modulo a constant

$$\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_3\rangle \propto \begin{cases} C_{123} \underbrace{(\langle \mathcal{O}_3\mathcal{O}_3\rangle + \text{descendants })}_{\text{fixed by conformal symmetry}} & \text{if } \mathcal{O}_3 \in \mathcal{O}_1 \times \mathcal{O}_2 \\ 0 & \text{otherwise} \end{cases}$$

Four point functions

Use OPE to reduce higher point functions to smaller ones

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \sim \sum_{\mathcal{O}}$$



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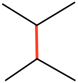

If OPE is associative, the two expansion must give the same result!

Definition of a CFT:

A Conformal Field Theory is an infinite set of primary operators $\mathcal{O}_{\Delta,\ell}$ and OPE coefficients C_{ijk} that satisfy crossing symmetry for all set of four-point functions.

Four point functions (more in details)

Recalling the OPE

$$\mathcal{O}(x_1) \times \mathcal{O}(x_2) = \sum_{\mathcal{O}'} \frac{C_{\mathcal{O}'}}{x_{12}^{2d-\Delta}} (\mathcal{O}'_{\Delta,\ell} + \text{descendants}) \qquad d = [\mathcal{O}]$$

Then

$$\langle \underbrace{\mathcal{O}(x_1)\mathcal{O}(x_2)}\underbrace{\mathcal{O}(x_3)\mathcal{O}(x_4)} \rangle = \frac{u^{-d}}{(x_{13}^{2d} x_{24}^{2d})} \sum_{O'_{\Delta,l}} C^2_{\mathcal{O}'} \underbrace{\left(\langle O'_{\Delta,\ell} O'_{\Delta,\ell} \rangle + \text{descendants}\right)}_{\text{function of } u,\, v \text{ only by conformal symmetry}}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

Conformal Blocks:

$$g_{\Delta,l}(u,v) \equiv \langle O'_{\Delta,\ell} O'_{\Delta,\ell} \rangle + \text{descendants}$$

They sum up the contribution of an entire representation

The Bootstrap program

Crossing Symmetry

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle \quad \text{vs} \quad \langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$$

They must produce the same result:

$$u^{-d}\left(1+\sum_{\Delta,l} C^2_{\Delta,\ell}\, g_{\Delta,\ell}(u,v)\right) = v^{-d}\left(1+\sum_{\Delta,\ell} C^2_{\Delta,\ell}\, g_{\Delta,l}(v,u)\right) \qquad d=[\mathcal{O}]$$

Crossing symmetry $\Rightarrow$ Sum Rule:

$$\sum_{\Delta,l} C^2_{\Delta,l} \underbrace{\frac{v^d g_{\Delta,\ell}(u,v) - u^d g_{\Delta,\ell}(v,u)}{u^d - v^d}}_{F_{d,\Delta,\ell}} = 1$$

[Rattazzi,Rychkov,Tonni, AV]

- Breakthrough in the field in 2000: first computation of $g_{\Delta,l}$ in D=2,4
- At present $g_{\Delta,\ell}$ are known numerically in any dimension
- Great efforts to extend to non scalar four point functions

Geometric interpretation

$$\sum_{\Delta,\ell} C^2_{\Delta,l} \begin{pmatrix} F_{d,\Delta,\ell} \\ \partial_u F_{d,\Delta,\ell} \\ \partial_v F_{d,\Delta,\ell} \\ \cdot \\ \cdot \\ \cdot \\ \partial_u^n \partial_v^m F_{d,\Delta,\ell} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

$F_{d,\Delta,\ell}$: combinations of conformal blocks
$n + m \leq N_{\max}$

1

Geometric interpretation

$$\sum_{\Delta,\ell} C^2_{\Delta,l} \begin{pmatrix} F_{d,\Delta,\ell} \\ \partial_u F_{d,\Delta,\ell} \\ \partial_v F_{d,\Delta,\ell} \\ \cdot \\ \cdot \\ \cdot \\ \partial_u^n \partial_v^m F_{d,\Delta,\ell} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

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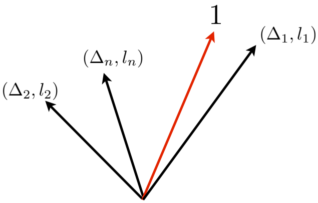

- All possible sums of vectors with positive coefficients define a cone

Geometric interpretation

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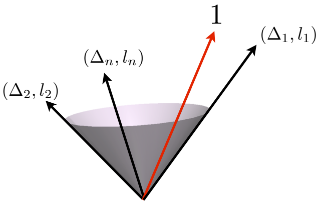

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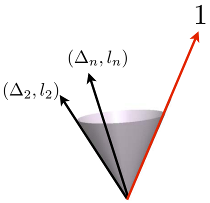

- All possible sums of vectors with positive coefficients define a cone
- Crossing symmetry satisfied ⇔ 1 is inside the cone
- Restrictions on the spectrum make the cone narrower

Geometric interpretation

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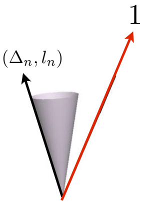

- All possible sums of vectors with positive coefficients define a cone
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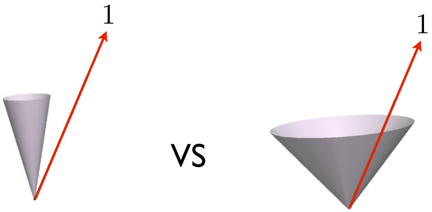
- Restrictions on the spectrum make the cone narrower
- A cone too narrow can't satisfy crossing symmetry: inconsistent spectrum

Geometric interpretation

How can we distinguish feasible spectra from unfeasible ones?



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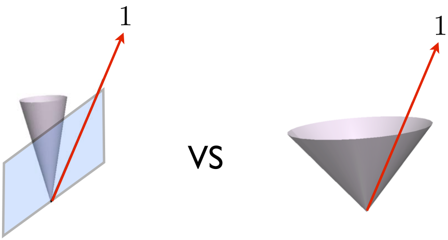


For unfeasible spectra it exists a plane separating the cone and the vector.

More formally...

Look for a Linear functional

$$\Lambda[F_{d,\Delta,\ell}] \equiv \sum_{n,m}^{N_{\max}} \lambda_{mn} \partial^n \partial^m F_{d,\Delta,\ell}$$

such that

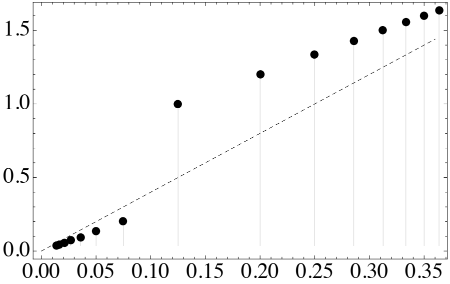
$$\Lambda[F_{d,\Delta,\ell}] > 0 \qquad \text{and} \qquad \Lambda[1] < 0$$

2D Example

Consider the OPE of scalar field in 2D CFT ϕ with itself:

$$\phi \times \phi \quad \sim \quad 1 + \phi^2 + \text{higher dimensional operators,}$$
$$+\text{higher spin operators} \qquad\qquad \Delta_\phi = [\phi],\ \Delta_{\phi^2} = [\phi^2]$$

What values of $(\Delta_\phi, \Delta_{\phi^2})$ are consistent with crossing symmetry?
(black points are minimal models, exactly known CFT's)



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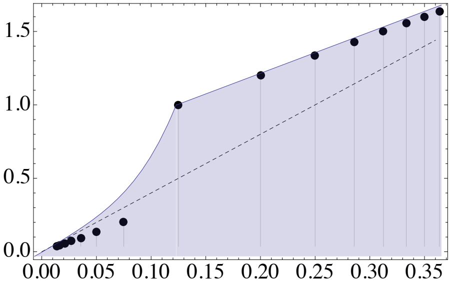
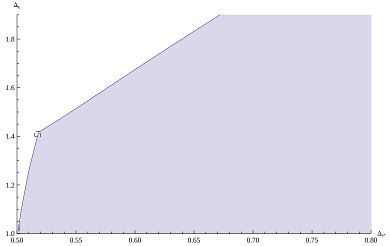


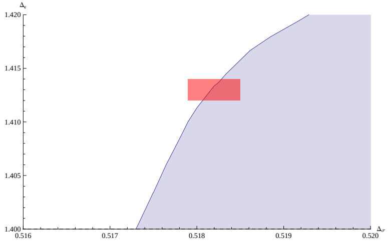
[Rychkov, AV]

3D Ising Model

Some notation: $\sigma \times \sigma \quad \sim \quad 1 + \epsilon + \epsilon' + \ldots$

Allowed regions in Δ_σ, Δ_ϵ plane ?





[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin, AV]

Already excluding part of $\epsilon-$expansion prediction

Going beyond: multiple correlators

So far we used a single four point function :$\langle\sigma\sigma\sigma\sigma\rangle$.

Let us include additional correlators: $\langle\epsilon\epsilon\epsilon\epsilon\rangle$, $\langle\sigma\epsilon\sigma\epsilon\rangle$.

Must consider all mixed OPE's :

$$\begin{array}{ccll} \sigma\times\sigma & \sim & 1+\epsilon+\epsilon'+.... & \mathbb{Z}_2-\text{even} \\ \sigma\times\epsilon & \sim & \sigma+\sigma'+.... & \mathbb{Z}_2-\text{odd} \\ \epsilon\times\epsilon & \sim & 1+\epsilon+\epsilon'+.... & \mathbb{Z}_2-\text{even} \end{array}$$

$$\langle\sigma(x_1)\epsilon(x_2)\sigma(x_3)\epsilon(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}} \lambda^2_{\sigma\epsilon\mathcal{O}}\tilde{g}_{\Delta,\ell}(u,v)$$

$$\langle\sigma(x_1)\epsilon(x_2)\sigma(x_3)\epsilon(x_4)\rangle \sim \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\sigma\sigma\mathcal{O}}\lambda_{\epsilon\epsilon\mathcal{O}}g_{\Delta,\ell}(u,v)$$

Second expansion is not a sum with positive coefficients: geometrical argument can't go through, but it can be generalized.

Study region allowed by multi-correlators crossing symmetry under the unique assumption that σ *and* ϵ *are the only two relevant scalar operators in theory.*

3D Ising Model: multiple correlators

Some notation:

$$\begin{aligned} \sigma \times \sigma &\sim 1 + \epsilon + \epsilon' + ... \quad \mathbb{Z}_2 - \text{even} \\ \sigma \times \epsilon &\sim \sigma + \sigma' + ... \quad \mathbb{Z}_2 - \text{odd} \\ \epsilon \times \epsilon &\sim 1 + \epsilon + \epsilon' + ... \quad \mathbb{Z}_2 - \text{even} \end{aligned}$$

Use $< \sigma\sigma\sigma\sigma >, < \sigma\sigma\epsilon\epsilon >, < \epsilon\epsilon\epsilon\epsilon >$

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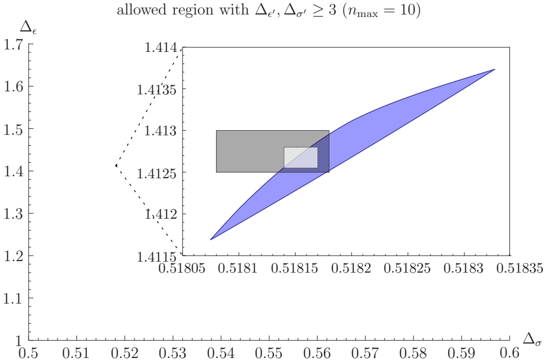
Use $<\sigma\sigma\sigma\sigma>, <\sigma\sigma\epsilon\epsilon>, <\epsilon\epsilon\epsilon\epsilon>$



[Kos,Poland,Simmons-Duffin]

3D Ising Model: multiple correlators

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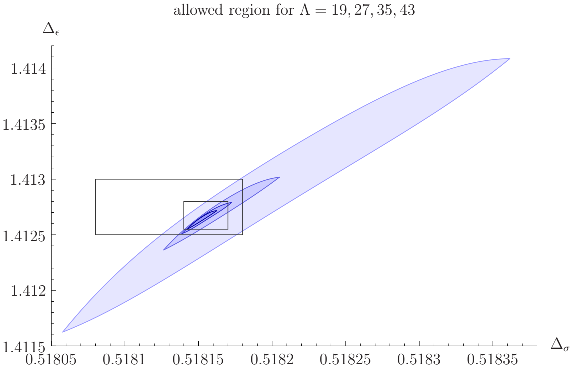

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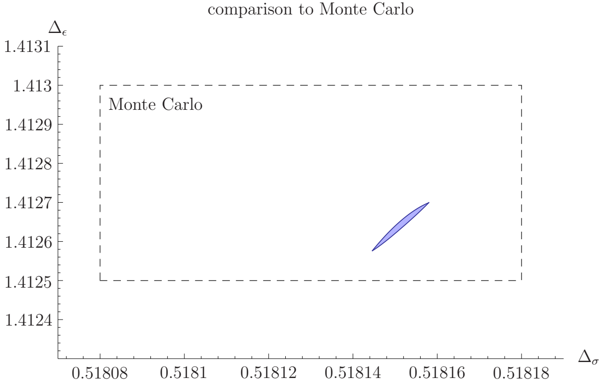


[Simmons-Duffin]

$$\Delta_\sigma \in [0.518145, 0.518157]$$

Summary

spin & $\mathbb{Z}_2$	name	Δ	OPE coefficient
$\ell = 0, \mathbb{Z}_2 = -$	σ	0.518145(6)	
$\ell = 0, \mathbb{Z}_2 = +$	ϵ ϵ'	1.41264(6) 3.8303(18)	$f^2_{\sigma\sigma\epsilon} = 1.10636(9)$ $f^2_{\sigma\sigma\epsilon'} = 0.002810(6)$
$\ell = 2, \mathbb{Z}_2 = +$	T T'	3 5.500(15)	$c_T/c_T^{\text{free}} = 0.946534(11)$ $f^2_{\sigma\sigma T'} = 2.97(2) \times 10^{-4}$

- The 3D Ising model is a CFT with only two relevant operators: σ and ϵ
- The 3D Ising model lies on the boundary of the region allowed by single correlator crossing symmetry
- Operator dimensions give the most precise determination of ν, η, ω critical exponents to date

$$\Delta_\sigma = 1/2 + \eta/2 \qquad \Delta_\epsilon = 3 - 1/\nu \qquad \Delta_{\epsilon'} = 3 + \omega \qquad \Delta_{\epsilon''} = 3 + \omega_2 \qquad \Delta_{\epsilon'''} = 3 + \omega_3$$

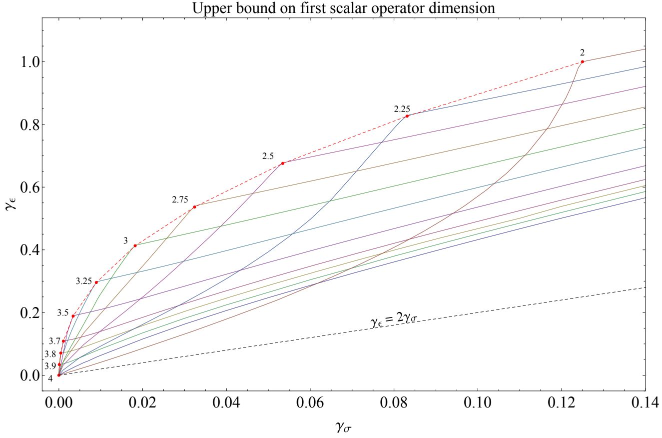
- First precise estimate of OPE coefficients and central charge
- Additional operators and coefficients (with larger errorbars) can be extracted
- What next? multiple correlators analysis can pinpoint the location of $O(N)$-models [F. Kos, D.Poland, D. Simmons-Duffin, AV, in progress]
- Study correlation functions containing conserved currents [AV, in progress ; M Costa & al, in progress]

BACKUP SLIDES

A proliferation of kinks

Compare bounds on the anomalous dimensions for various space-time dimensions D:

$$\gamma_\sigma = \Delta_\sigma - \frac{(D-2)}{2} \qquad \gamma_\epsilon = \Delta_\epsilon - (D-2)$$

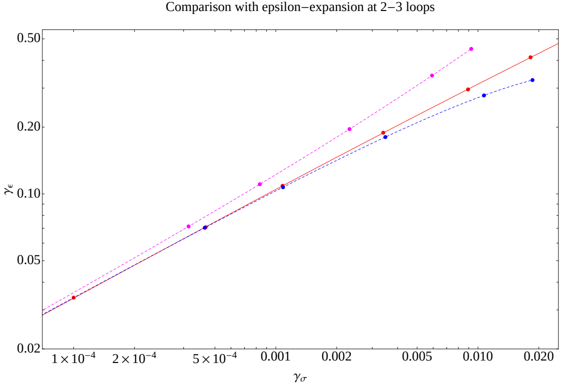


[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin, AV]

Epsilon Expansion: $D = 4 - \varepsilon$

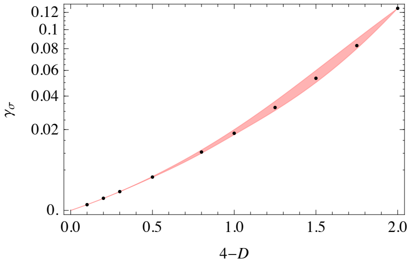
$$\gamma_\sigma = \frac{(N+2)\varepsilon^2}{4(N+8)^2} + O(\varepsilon^3)$$

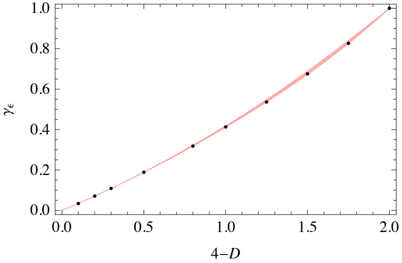
$$\gamma_\epsilon = \frac{(N+2)\varepsilon}{N+8} + \frac{(N+2)(13N+44)\varepsilon^2}{2(N+8)^3} + O(\varepsilon^3)$$



- Kinks from previous slide
- $O(\varepsilon^2)$
- $O(\varepsilon^3)$

Epsilon Expansion: $D = 4 - \varepsilon$





- Our prediction (points)
- Borel resumed series: central values and errors (bands)
  [Guillou,Zinn-Justin]

Multiple correlators

When using multiple correlators the search for linear functionals must be modified to accommodate non squared OPE coefficients:

Single correlator:

$$\sum_{\mathcal{O}_{\Delta,\ell}} \lambda^2_{\sigma\sigma\mathcal{O}} F_{\Delta_\sigma,\Delta,\ell} = 1$$

Look for functional

$$\Lambda[F_{\Delta_\sigma,\Delta,\ell}] \equiv \sum_{n,m}^{N_{\max}} \lambda_{mn} \partial^n \partial^m F_{\Delta_\sigma,\Delta,\ell}$$

such that:

$$\Lambda[F_{\Delta_\sigma,\Delta,\ell}] > 0 \qquad \text{and} \qquad \Lambda[1] < 0$$

Multi correlators:

$$\sum_{\mathcal{O}_{\Delta,\ell}} \vec{\lambda}^T_{\mathcal{O}} M_{\Delta,\ell} \vec{\lambda}_{\mathcal{O}} + \sum_{\mathcal{O}'_{\Delta,\ell}} \lambda^2_{\sigma\epsilon\mathcal{O}'} \widetilde{F}_{\Delta,\ell} = 0$$

$$M_{\Delta,\ell} = \begin{pmatrix} 0 & \frac{1}{2} F_{\Delta_\sigma,\Delta,\ell} \\ \frac{1}{2} F_{\Delta_\epsilon,\Delta,\ell} & 0 \end{pmatrix}, \; \vec{\lambda}_{\mathcal{O}} = \begin{pmatrix} \lambda_{\sigma\sigma\mathcal{O}} \\ \lambda_{\epsilon\epsilon\mathcal{O}} \end{pmatrix}$$

Look for a functional acting on matrices

$$\Lambda[M_{\Delta,\ell}] \equiv \sum_{n,m}^{N_{\max}} \lambda_{mn} \partial^n \partial^m M_{\Delta,\ell}$$

such that (semidefinite positiveness condition)

$$\Lambda[M_{\Delta,\ell}] \succeq 0 \qquad \text{and} \qquad \Lambda[M_{0,0}] < 0$$

Solving Ising 3D: boundary spectrum

- On the boundary of the allowed region the solution to crossing is unique: the whole spectrum and OPE coefficients can be reconstructed.
- Assuming to leave on the upper boundary of the allowed island (note that increasing the numerical power it is approximatively stable)

Recall OPE: $\sigma \times \sigma \sim 1 + \epsilon + \epsilon' + \ldots\ (\ell{=}0)$

$$+ T_{\mu\nu} + T'_{\mu\nu} + \ldots..\ (\ell{=}2)$$
$$+ \ldots..\ (\ell > 2)$$

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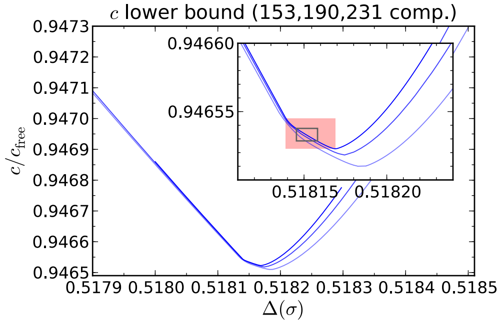
$+ \ldots..$ ($\ell >$2)

Prediction for central charge:



$c/c_{\text{free}} \in [0.946528, 0.946538]$

red rectangle : assuming Ising 3D has minimal central charge

gray rectangle: multiple correlators

[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin, AV]

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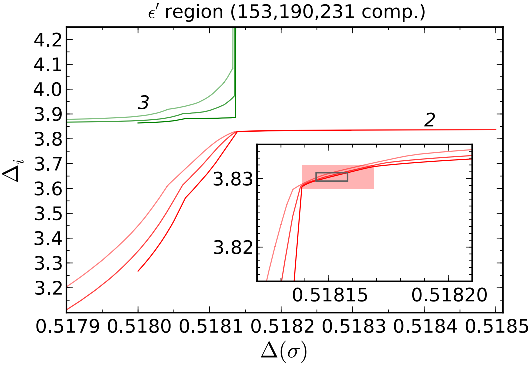
$+\, T_{\mu\nu} + T'_{\mu\nu} + \ldots..\ (\ell{=}2)$
$+\, \ldots..\ (\ell > 2)$

Prediction for ϵ':



$\Delta_{\epsilon'} \in [3.829, 3.831]$

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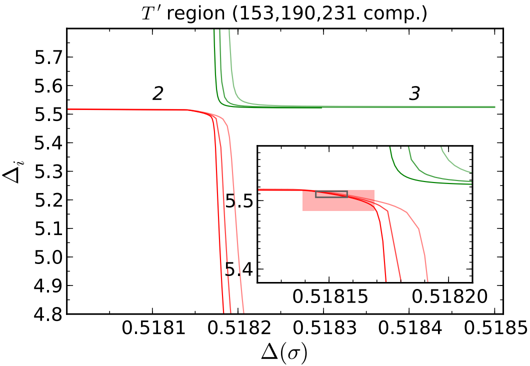
[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin, AV]

Solving Ising 3D: boundary spectrum

- On the boundary of the allowed region the solution to crossing is unique: the whole spectrum and OPE coefficients can be reconstructed.
- Assuming to leave on the upper boundary of the allowed island (note that increasing the numerical power it is approximatively stable)

Recall OPE: $\sigma \times \sigma \sim 1 + \epsilon + \epsilon' + \ldots\ (\ell{=}0)$
$+ T_{\mu\nu} + T'_{\mu\nu} + \ldots..\ (\ell{=}2)$
$+ \ldots..\ (\ell > 2)$

Prediction for T':



$\Delta_{T'} \in [5.505, 5.515]$

red rectangle : assuming Ising 3D has minimal central charge

gray rectangle: multiple correlators

[El-Showk,Paulos,Poland,Rychkov,Simmons-Duffin, AV]